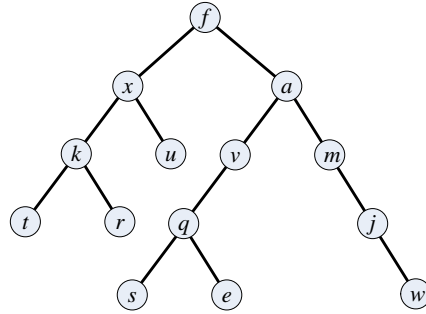


The point values for each question is given within []. The total number of points for this assignment is 30.

- [6] 1. Provide the pre-, in-, and postorder traversals for the following tree:



2. Consider the following table of letter frequencies for a particular data set:

Letter	Frequency	Code
A	15	
E	25	
I	10	
O	30	
U	15	
Y	5	

- [5] (a) Construct a Huffman tree to be used to derive a minimal prefix code for the letters.

- [3] (b) Complete the last column in the table with the bitstrings for the prefix code derived from your Huffman tree.

- [3] 3. For each of the following mathematical structures circle G if the mathematical structure is a group, M if it is just a monoid, or N if it is neither a group nor a monoid.
- (a) G M N  $(\mathbb{R}, +)$
  - (b) G M N  $(\mathbb{Z}, \cdot)$
  - (c) G M N  $(\mathbb{N}, -)$
4. Determine which of the following mathematical structures are groups. For a group, you need to show closure, associativity, identity, and invertibility; otherwise, you need only show that one of these properties does not hold.
- [3] (a)  $(\{-1, 1\}, \cdot)$ , where  $\cdot$  is normal multiplication.
- [3] (b)  $(\mathbb{Z}, \diamond)$ , where  $a \diamond b$  is the larger of  $a$  and  $b$ .
- [3] 5. Show that the set of even integers form a subgroup of  $(\mathbb{Z}, +)$ .
6. Consider the monoid  $M_1 = (\mathbb{Z}, +)$ , where  $+$  is normal integer addition, and the monoid  $M_2 = (A, +)$ , where  $A$  is the set of  $2 \times 2$  integer matrices and  $+$  is normal matrix addition. Next consider the function  $f : M_1 \rightarrow M_2$ , such that
- $$f(x) = \begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix}.$$
- [3] (a) Show that  $f$  is a homomorphism from  $M_1$  to  $M_2$ .
- [1] (b) Is  $f$  an isomorphism?