MATH 280 Discrete Mathematical Structures Assignment #7

Name ____________________

The point values for each question is given within []. The total number of points for this assignment is 43.

[15] 1. For each of the following relations on the set $\{1,2,3,4\}$, write **R** if the relation is reflexive, **S** if the relation is symmetric, **A** if the relation is antisymmetric, and **T** if the relation is transitive.

(a) $\{(1,2),(2,3),(3,4)\}$

(b) $\{(1,1),(1,2),(2,1),(2,2),(3,3),(4,4)\}$

(c) $\{(2,2),(2,3),(2,4),(3,2),(3,3),(3,4)\}$

(d) $\{(1,3),(1,4),(2,3),(2,4),(3,1),(3,4)\}$

(e) $\{(1,1),(2,2),(3,3),(4,4)\}$

[12] 2. For each of the following relations on the set of integers, $\mathbb{Z}$, write **R** if the relation is reflexive, **S** if the relation is symmetric, **A** if the relation is antisymmetric, and **T** if the relation is transitive.

(a) $x \ R \ y \leftrightarrow xy \geq 1$

(b) $x \ R \ y \leftrightarrow \exists k \in \mathbb{Z}$ such that $x = ky$

(c) $x \ R \ y \leftrightarrow x = y+1$ or $x = y-1$

(d) $x \ R \ y \leftrightarrow x \geq y^2$

[6] 3. Draw a Hasse diagram for each of the following partially ordered sets.

(a) $S = \{2,4,5,10,12,20,25\}$, $x \ R \ y \leftrightarrow x$ divides y (said another way, x is a factor of y)

(b) $T = \wp(\{1,2,3\})$, $A \ R \ B \leftrightarrow A \subseteq B$

[4] 4. Find the maximal and minimal elements of

(a) the poset described in Question 3a

(b) the poset described in Question 3b

[6] 5. In the C++ programming language, character strings can be represented by objects of type `std::string`. If `s` is a `std::string` object, `s.length()` is the number of characters that make up `s`. All strings in C++ have a finite length. For all integers $0 \leq i <$ `s.length()`, the expression `s[`i`]` evaluates to the character at position i in string `s`. As in all C-derived languages, the first character in string `s` is `s[0]`.

(a) Let R be the relation on the set of all C++ strings such that $s \ R \ t \leftrightarrow s$`.length()` $= t$`.length()`. Prove that R is an equivalence relation on the set of all C++ strings.

(b) Let R be the relation on the set of all non-empty C++ strings such that $s \ R \ t \leftrightarrow s$`[0]` $= t$`[0]`. Prove that R is an equivalence relation on the set of all non-empty C++ strings.